

#23

$$\sec^4 x + \tan^4 x$$

$$(\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$$

$$1 (\sec^2 x + \tan^2 x)$$

$$\sec^2 x + \tan^2 x$$

Look at

$$a^4 - b^4 = (a^2 - b^2)(a^2 + b^2)$$

$$1 + \tan^2 u = \sec^2 u$$

$$\text{So } 1 = \sec^2 u - \tan^2 u$$

#33

$$\frac{1 - \sin^2 x}{\csc^2 x - 1} = \frac{\cos^2 x}{\cot^2 x}$$

$$= \frac{\cos^2 x}{\frac{\cos^2 x}{\sin^2 x}} \rightarrow \div$$

$$= \cancel{\cos^2 x} \cdot \frac{\sin^2 x}{\cancel{\cos^2 x}}$$

$$= \sin^2 x$$

Write  
right  
now

#34

$$\frac{1}{\tan^2 x + 1} = \frac{1}{\sec^2 x}$$
$$= \cos^2 x$$

identity  
rec. function

#35

$$\frac{\sec \alpha (\sin \alpha)}{\tan \alpha}$$

$$= \frac{\frac{1}{\cos \alpha} (\sin \alpha)}{\tan \alpha}$$

$$= \frac{\tan \alpha}{\tan \alpha}$$

$$= 1$$

$$\begin{aligned} \#39 \quad \frac{\cos^2 y}{1 - \sin y} &= \frac{1 - \sin^2 y}{1 - \sin y} && \text{Identity} \\ &= \frac{(\cancel{1 - \sin y})(1 + \sin y)}{(\cancel{1 - \sin y})} && \text{Factor} \\ &= 1 + \sin y && \text{eliminate} \\ &&& \text{common} \\ &&& \text{factors} \end{aligned}$$

$$\#41 \quad \sin \beta + \tan \beta + \cos \beta$$

$$= \sin \beta \frac{\sin \beta}{\cos \beta} + \cos \beta \quad \text{Identity}$$

$$= \frac{\sin^2 \beta}{\cos \beta} + \cos \beta \left( \frac{\cos \beta}{\cos \beta} \right) \quad \begin{array}{l} \text{Simplify} \\ \text{Common den.} \end{array}$$

$$\text{simplify} = \frac{\sin^2 \beta + \cos^2 \beta}{\cos \beta}$$

Identity

Identity

$$= \frac{1}{\cos \beta} \\ = \sec \beta$$

#44

$$\sin \theta \sec \theta + \cos \theta \csc \theta$$

$$\sin \theta \frac{1}{\cos \theta} + \cos \theta \frac{1}{\sin \theta}$$

$$\left( \frac{\sin \theta}{\sin \theta} \right) \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \left( \frac{\cos \theta}{\cos \theta} \right)$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}$$

$$\frac{1}{\cos \theta \sin \theta}$$

$$\sec \theta \csc \theta$$

(conjugates) Factoring

Examples

$$\frac{\sec^2 \theta - 1}{\sec \theta - 1}$$

$$= \frac{(\cancel{\sec \theta - 1})(\sec \theta + 1)}{(\cancel{\sec \theta - 1})}$$

$$= \sec \theta + 1$$

Example #2

$$\frac{4 + \tan^2 \theta + \tan \theta - 3}{\tan \theta + 1}$$

$$\frac{(4 + \tan \theta - 3)(\cancel{\tan \theta + 1})}{\cancel{\tan \theta + 1}}$$

$$4 + \tan \theta - 3$$

p. 228 (45-54)

$$4x^2 + x - 3$$
$$(4x - 3)(x + 1)$$